QUIZ #3 – Solutions

Values of each problem:
#1 - 5 points, #2 - 5 points, #3 - 1 point, #4 - 1 point, #5 - 3 points
15 points total

1. 5 points

Since $dy/dt = 2x + 4$ and $dx/dt = (3 - y)/2$, it follows that

$$\frac{dy}{dx} = \frac{2x+4}{(3-y)/2} \quad \Longrightarrow \quad (3-y)\,dy = 4(x+2)\,dx,$$

a separated equation. A one-parameter family of solutions is defined implicitly by

$$3y - \frac{y^2}{2} = 4\left(\frac{x^2}{2} + 2x\right) + C.$$

Since the electron passes through $(0, 3)$, we must have $9 - 9/2 = C$, and therefore

$$3y - \frac{y^2}{2} = 2x^2 + 8x + \frac{9}{2} \quad \Longrightarrow \quad 4x^2 + y^2 + 16x - 6y + 9 = 0.$$

This is an ellipse.

2. 5 points

Since $2y_1'' - 16y_1' + 32y_1 = 96e^{4x} - 192e^{4x} + 96e^{4x} = 0$, and

$$2y_2'' - 16y_2' + 32y_2 = 2(-32xe^{4x} - 16e^{4x}) - 16(-8xe^{4x} - 2e^{4x}) + 32(-2xe^{4x}) = 0,$$

$y_1(x)$ and $y_2(x)$ are solutions of the equation. Because the equation is linear and homogeneous, a general solution is $y(x) = D_1(3e^{4x}) + D_2(-2xe^{4x}) = (C_1 + C_2x)e^{4x}$.

3. 1 point

$(-2 + i)(3 - 4i) = -6 + 3i + (-2)(-4)i - 4i^2 = -6 + 11i + 4 = -2 + 11i$

4. 1 point

$$\begin{aligned}(1 - i) / (3 + 2i) &= (1 - i)(3 - 2i) / (3 + 2i)(3 - 2i) \\ &= (3 - 3i - 2i + 2i^2) / (3^2 + 2^2) \\ &= (1 - 5i) / 13 \\ &= (1/13) + (-5/13)i\end{aligned}$$

5. 3 points

$4x^2 - 2x + 5 = 0 \quad \Leftrightarrow \quad x^2 - x/2 + 5/4 = 0$

$\Leftrightarrow \quad (x - 1/4)^2 + (5/4 - 1/16) = (x - 1/4)^2 + (-19/16) = 0$

$\Leftrightarrow \quad x - 1/4 = \sqrt{-19}/4 \quad$ or $\quad x - 1/4 = -\sqrt{-19}/4$

$\Leftrightarrow \quad x = (1/4) + (\sqrt{19}/4)\,i \quad$ or $\quad x = (1/4) - (\sqrt{19}/4)\,i$